International Journal of Theoretical Physics, Vol. 42, No. 12, December 2003 (© 2003)

Mass Generation From Lie Algebra Extensions

R. Aldrovandi,[1,3] V. C. de Andrade,[2] A. L. Barbosa,[1] and J. G. Pereira[1]

Received May 25, 2003

Applied to the electroweak interactions, the theory of Lie algebra extensions suggests a mechanism by which the boson masses are generated without resource to spontaneous symmetry breaking. It starts from a gauge theory without any additional scalar field. All the couplings predicted by the Weinberg–Salam theory are present, and a few others which are nevertheless consistent within the model.

KEY WORDS: geometry; gauge fields; algebra extensions.

1. INTRODUCTION

In the Weinberg–Salam model, the boson masses are generated by spontaneous breakdown of the gauge symmetry, a process which hides invariance behind the scene. In pure gauge theories like QED and QCD, gauge invariance is manifest in the forefront and there are no massive bosons.

The geometrical background of pure gauge theories is well known: a principal bundle with space-time for base space and the gauge group as structure group (Aldrovandi and Pereira, 1995; Kobayashi and Nomizu, 1963; Nakahara, 1990). An essential feature of this structure is its direct product character: The bundle is locally trivial. This is to say that the fundamental vector fields (that represent the group generators on the bundle) commute with the horizontal-lift vector fields (that represent a space-time basis). Or still, that the l-form (the connection) that takes the fundamental fields back to the corresponding generators in the group Lie algebra belongs to the adjoint representation of the group. Connections appear as interaction-mediating vector fields in Field Theory: as the photon field (potential four-vector) in QED, as the gluon fields in standard QCD. Such nonmassive

[1] Instituto de Física Teórica, Universidade Estadual Paulista, São Paulo SP, Brazil.
[2] Instituto de Física, Universidade de Brasília, Brasília DF, Brazil.
[3] To whom correspondence should be addressed at Instituto de Física Teórica, Universidade Estadual Paulista, Rua Pamplona 145, 01405900 São Paulo SP, Brazil; e-mail: ra@ift.unesp.br.

boson fields transform indeed like connections, and therefore preserve the gauge invariance of the theory.

Electroweak theory, with its massive bosons, seems quite different from QED and QCD from the geometrical point of view. The massive mediating fields do not transform like connections—they actually do not transform at all. A natural question to be asked is: *What is the geometrical background of electroweak theory?* An effort to unveil the underlying geometry of electroweak interactions was initiated years ago (Aldrovandi, 1991b). It was found that adding a noncovariant piece to a connection led to field equations hinting both at the possibility of mass generation and at effects of gravitational character. Some more steps in that way have been taken recently (Aldrovandi and Barbosa, 2000), mainly in what concerns the gravitational aspects.

The aim of this paper is to present an electroweak model whose background geometry generates by itself the boson masses, with no appeal to spontaneous breaking. We intend, of course, to remain as near as possible to the Weinberg–Salam (W-S) theory because of its overwhelmingly successful phenomenology (Cheng and Li, 1984; Greiner and Müller, 1996). Our model also starts from a true gauge theory, but does without the additional scalar field and its vacuum-modifying potential. The basic point is that the algebra of vector fields tangent to its principal bundle encapsulates the whole local geometry of a gauge theory. The new background referred to (that we call an "extended gauge model") is obtained when that algebra is conveniently modified by a procedure described by the theory of Lie algebra extensions (Aldrovandi, 1991a). In the electroweak case, it is applied to the Glashow algebra (GA), a Lie algebra obtained from the generators of $SU(2) \otimes U(1)$, but which incorporates the mixing angles in its structure coefficients. The resulting model has all the couplings present in the W-S model, but also some which are absent.

The extended gauge formalism is presented in section 2. That it does lead to mass generation for the electroweak bosons is shown in section 4, but section 3 gives a necessary, preparatory exposition of the Glashow algebra, which is the Lie algebra of the real structure group of the model (Glashow group) (Barbosa, 2000). Some general considerations on the model, including the problems still under investigation, are made in the final section.

2. EXTENDED GAUGE THEORIES

Let us begin by presenting an extended gauge model. In general terms, a connection

$$A_\mu = A^a{}_\mu X_a \tag{1}$$

defines the covariant derivative

$$X_\mu = \partial_\mu - A_\mu, \tag{2}$$

where $\{X_a\}$ are the fundamental fields, and $\{X_\mu\}$ the horizontal-lift fields (Cho, 1975). A general basis on the fiber bundle is defined by the set $\{X_a, X_\mu\}$. The geometric setup is *locally* characterized by the corresponding commutation relations,

$$\begin{aligned} [X_\mu, X_\nu] &= -F^a{}_{\mu\nu} X_a, \\ [X_a, X_\mu] &= 0, \\ [X_a, X_b] &= f^c{}_{ab} X_c. \end{aligned} \tag{3}$$

The second commutator above declares the direct product character of the bundle geometry and, furthermore, enforces the necessary adjoint behavior of the connection A:

$$X_a\left(A^b{}_\mu\right) = f^b{}_{ca} A^c_\mu. \tag{4}$$

Notice that we are working on the bundle manifold. The usual (nonhomogeneous) derivative term in the transformation of $A^a{}_\mu$ only turns up when the expression above is pulled back to space-time (Popov, 1975). From the middle commutator in (3) and (4) we obtain the expression for the field strength,

$$F^a{}_{\mu\nu} = \partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu + f^a{}_{bc} A^b{}_\mu A^c{}_\nu. \tag{5}$$

One of the Jacobi identities for the set of commutators gives

$$X_a\left(F^b{}_{\mu\nu}\right) = f^b{}_{ca} F^c{}_{\mu\nu}. \tag{6}$$

This condition shows that F also belongs to the adjoint representation of the gauge group, whose generators are represented, on the bundle, by the fields X_a. In the language of Lie algebra extensions, F is called the "nonlinearity indicator." In the direct product case, it coincides with the field strength of the gauge field.

The Jacobi identity for three fields X_μ, X_ν, X_ρ, gives rise to the Bianchi identity. Gauge field dynamics can be obtained by using the duality prescription: The sourceless field equations are written just as the Bianchi identity, but applied to the dual of the field strength. The Yang–Mills equations come out:

$$X_\mu F^{a\mu\nu} = 0. \tag{7}$$

An extended gauge theory comes forth when we break the direct product in (3) through a change of basis (Aldrovandi and Barbosa, 2000),

$$X'_\mu = X_\mu - B^a{}_\mu X_a, \tag{8}$$

that is equivalent to

$$X'_\mu = \partial_\mu - A'^a{}_\mu X_a \tag{9}$$

if we define

$$A'^a{}_\mu \equiv A^a{}_\mu + B^a{}_\mu. \tag{10}$$

The 1-form $A'^a{}_\mu$ can be seen as a connection deformed by the addition of a noncovariant form $B^a{}_\mu$ (Aldrovandi, 1991b). Since the direct product is intimately related to the adjoint behavior of a connection, it is necessary that neither $A'^a{}_\mu$ nor $B^a{}_\mu$ belong to the adjoint representation. Expression (9) leads to commutation relations of the form

$$[X'_\mu, X'_\nu] = -F'^a{}_{\mu\nu} X_a,$$
$$[X_a, X'_\mu] = C^c{}_{a\mu} X_c,$$
$$[X_a, X'_b] = f^c{}_{ab} X_c. \tag{11}$$

The second commutator above gives the transformation law for the object $A'^a{}_\mu$ under the action of the gauge group:

$$X_b\left(A'^a{}_\mu\right) = f^a{}_{cb} A'^c{}_\mu - C^a{}_{b\mu}. \tag{12}$$

Comparison with (4) shows that $C^a{}_{b\mu}$ is the measure of its deviation from covariant behavior. We shall call the derivative (9), with the noncovariant $A'^a{}_\mu$ in the position of a connection, a *generalized derivative*.

The behavior of $B^a{}_\mu$, under the group action is obtained by using Eq. (8) in the second commutator of Eqs. (11):

$$X_b\left(B^a{}_\mu\right) = f^a{}_{cb} B^c_\mu - C^a{}_{b\mu}. \tag{13}$$

The new nonlinearity indicator $F'^a{}_{\mu\nu}$ can be obtained from the first commutator:

$$F'^a{}_{\mu\nu} = \partial_\mu A'^a{}_\nu - \partial_\nu A'^a{}_\mu + f^a{}_{bc} A'^b{}_\mu A'^c{}_\nu - C^a{}_{c\mu} A'^c{}_\nu + C^a{}_{c\nu} A'^c{}_\mu. \tag{14}$$

Its behavior under the group action is fixed by the Jacobi identity for two fields X'_μ and one field X_a:

$$X_b F'^a{}_{\mu\nu} = f^a{}_{cb} F'^c{}_{\mu\nu} - R'^a{}_{b\mu\nu}, \tag{15}$$

where

$$R'^a{}_{b\mu\nu} = X'_\mu C^a{}_{b\nu} - X'_\nu C^a{}_{b\nu} - C^a{}_{d\mu} C^d{}_{b\nu} + C^a{}_{d\nu} C^d{}_{b\nu}. \tag{16}$$

Dynamics associated with algebra (11) is obtained by applying the duality prescription to the Jacobi identity involving three fields X'_μ. The field equations turn out to be

$$X'_\mu F'^{a\mu\nu} - C^a{}_{d\mu} F'^{d\mu\nu} = 0. \tag{17}$$

These equations are, of course, linked to the choice of C, which is constrained by a Jacobi identity for two fields X_a and a field X'_μ:

$$X_a\left(C^c{}_{b\mu}\right) - X_b\left(C^c{}_{a\mu}\right) + f^c{}_{bd} C^d{}_{a\mu} - f^d{}_{ba} C^c{}_{d\mu} - f^c{}_{ad} C^d{}_{b\mu} = 0. \tag{18}$$

The term $R'^{a}{}_{b\mu\nu}$, which breaks the covariance of $F'^{a}{}_{\mu\nu}$, is constrained by an equation similar to the one satisfied by C:

$$X_a\left(R^c{}_{b\mu\nu}\right) - X_b\left(R^c{}_{a\mu\nu}\right) + f^c{}_{ad}R^d{}_{b\mu\nu} + f^d{}_{ba}R^c{}_{d\mu\nu} - f^c{}_{bd}R^d{}_{a\mu\nu} = 0. \quad (19)$$

We can infer by introducing (14) and (15) in (17) that a mass term for A' can appear, a fact we shall make profit of in section 4. Thus, the basis change that breaks the direct product can lead to a theory with massive vector fields which, of course, behave no more like connections. This is what happens in the W-S model, here achieved through a different process.

The formalism just presented can be applied to any gauge group. Besides the same nonlinearity indicator $F'^{a}{}_{\mu\nu}$ appearing in (11), an extra nonlinear term $R'^{c}{}_{a\mu\nu}$ turns up, whose aspect (16) suggests a curvature. With some further elaboration it does lead to models of gravitational type for 4-dimensional groups (Aldrovandi and Barbosa, 2000), but that will not be our concern here. The counting of the degrees of freedom of the theory will be discussed at the end of section 4.

3. GLASHOW ALGEBRA AND THE ELECTROWEAK INTERACTION

In this section we present some basic concepts concerning the Glashow algebra (Barbosa, 2000). The main objective is to give a geometrical role to the mixing angle in the electroweak theory. The Glashow algebra is constructed as an extra support to the gauge theory, since the introduction of the mixing angle emerges naturally in their structure constants. In the usual W-S approach, the mixing angle is introduced in order to diagonalize the mass matrix, the physical fields appear as combinations of the original gauge potentials and the underlying gauge algebra remains unchanged.

We start by considering a gauge theory and the direct product $SU(2) \otimes U(1)$,

$$[X_a, X_b] = \epsilon^c{}_{ab}X_c, \quad \text{for } a, b = 1, 2, 3 \quad (20)$$

$$[X_a, X_b] = 0, \quad \text{for } a \text{ or } b = 0. \quad (21)$$

The direct product $SU(2) \otimes U(1)$ leads to a sum of two gauge theories, one abelian ($f^a{}_{bc} = 0$) and the other with gauge potentials $A^a{}_\mu$, and field strength given by (5), satisfying respectively the transformations properties (4) and (6) with structure constants $f^a{}_{bc} = \epsilon^a{}_{bc}$. The abelian and nonabelian sectors are quite independent. Furthermore, there are no charged fields in the algebraic schemes presented in section 2. We know, however, from the experimental data that there are two charged bosons W^+ and W^-, and also that there is a mixture between the abelian and nonabelian sectors giving an essential contribution to the electron–positron cross section (Greiner and Müller, 1996; Mandl and Shaw, 1984). Thus, two charged vector fields must be constructed and the algebra underlying the theory must be modified to produce the necessary mixing.

Let us denote by $\{\bar{X}_a\}$ a basis for the generators of the new algebra, in terms of which the physical gauge potentials will be written $\bar{A}_\mu = \bar{A}^a{}_\mu \bar{X}_a$. Since this time the change is only in the internal sector, we must impose the following condition:

$$\bar{A}_\mu = \bar{A}^a{}_\mu \bar{X}_a = A^a{}_\mu X_a = A_\mu. \tag{22}$$

That is, the space-time sector remains unchanged. Two charged gauge fields and two neutral fields are constructed as a linear combinations of the original ones:

$$\bar{A}^1{}_\mu = \frac{1}{\sqrt{2}} \left(A^1{}_\mu - i A^2{}_\mu \right), \tag{23}$$

$$\bar{A}^2{}_\mu = \frac{1}{\sqrt{2}} \left(A^1{}_\mu + i A^2{}_\mu \right), \tag{24}$$

$$\bar{A}^0{}_\mu = \sin\theta A^3{}_\mu + \cos\theta A^0{}_\mu, \tag{25}$$

$$\bar{A}^3{}_\mu = \cos\theta A^3{}_\mu - \sin\theta A^0{}_\mu, \tag{26}$$

where θ is a mixing angle. Using Eqs. (22) and (23)–(26) we obtain the GA generators in terms of those of the direct product $SU(2) \otimes U(1)$:

$$\bar{X}_1 = \frac{1}{\sqrt{2}} (X_1 - i X_2), \tag{27}$$

$$\bar{X}_2 = \frac{1}{\sqrt{2}} (X_1 + i X_2), \tag{28}$$

$$\bar{X}_3 = \cos\theta X_3 - \sin\theta X_0, \tag{29}$$

$$\bar{X}_0 = \sin\theta X_3 + \cos\theta X_0. \tag{30}$$

The structure constants $\bar{f}^c{}_{ab}$ of the Glashow algebra

$$[\bar{X}_a, \bar{X}_b] = \bar{f}^c{}_{ab} \bar{X}_c \tag{31}$$

are

$$\begin{aligned} \bar{f}^0{}_{12} &= -i \sin\theta, \quad & \bar{f}^3{}_{12} &= -i \cos\theta, \\ \bar{f}^1{}_{10} &= +i \sin\theta, \quad & \bar{f}^1{}_{13} &= +i \cos\theta, \\ \bar{f}^2{}_{23} &= -i \cos\theta, \quad & \bar{f}^2{}_{20} &= -i \sin\theta. \end{aligned} \tag{32}$$

The mixture is in this way incorporated in the algebra through the structure constants. The determinant of the related Killing–Cartan bilinear form,

$$g_{ab} = \bar{f}^c{}_{ad} \bar{f}^d{}_{bc}, \tag{33}$$

is equal to zero, characterizing GA as a nonsemisimple algebra. It can be shown that the physical fields (without mass), two charged and two neutral, are indeed gauge fields, that is, they transform like connections by the action of the group generators $\bar{X}_a$.

The field strength associated to the physical fields is constructed by appealing to the same arguments preceding Eq. (22). By writing

$$\bar{F}^a{}_{\mu\nu}\bar{X}_a = F^a{}_{\mu\nu}X_a, \tag{34}$$

using Eqs. (5) and (22), and introducing the coupling constant g, we arrive at

$$\bar{F}^a{}_{\mu\nu} = g\left[\partial_\mu \bar{A}^a{}_\nu - \partial_\nu \bar{A}^a{}_\mu + g\,\bar{f}^a{}_{bc}\bar{A}^b{}_\mu \bar{A}^c{}_\nu\right]. \tag{35}$$

Expression (35) reflects a crucial result that we shall explore from now on: All the expressions presented in section 2 are valid if we replace A by $\bar{A}$, and the structure constants f by $\bar{f}$.

The importance of GA is corroborated by the following example: It is possible to obtain from it the correct Lagrangian for the massless electroweak theory. This comes out by taking the usual gauge Lagrangian

$$L = \frac{1}{8g^2}\int d^3x\ \mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) = \frac{1}{8g^2}\int d^3x\,\bar{F}^a{}_{\mu\nu}\bar{F}^{b\mu\nu}\ \mathrm{tr}(\bar{X}^*_a\bar{X}^*_b), \tag{36}$$

with a representation $\{\bar{X}^*_a\}$ whose nonvanishing traces (Barbosa, 2000) are given by:

$$\mathrm{tr}(\bar{X}^*_a\bar{X}^*_b) = -2, \quad \text{for } (a,b) = (0,0), (1,2), (2,1)\, e\, (3,3). \tag{37}$$

The correct Lagrangian (Mandl and Shaw, 1984) appears after making the following associations:

$$\begin{aligned}
\bar{A}^1{}_\nu &\to W^-_\nu,\\
\bar{A}^2{}_\nu &\to W^+_\nu,\\
\bar{A}^3{}_\nu &\to Z_\nu,\\
\bar{A}^0{}_\nu &\to A_\nu.
\end{aligned} \tag{38}$$

Though we have presented the Lagrangian for the massless electroweak theory, we proceed using the formalism of equations of motion since the theory of extended Lie algebras works directly with that formalism, as can be seen from Eq. (17).

4. GENERATION OF MASS

4.1. Equation of Motion

As seen in section 2 an extended gauge theory can be obtained by adding to a connection a noncovariant part:

$$\bar{A}'^{c}{}_{\mu} = \bar{A}'^{a}{}_{\mu} + \bar{B}^{a}{}_{\mu}.$$

$\bar{A}'^{c}{}_{\mu}$ will be interpreted as the physical massive fields, that is, they will be identified as

$$\begin{aligned}\bar{A}'^{1}{}_{\nu} &\to W^{-}_{\nu},\\ \bar{A}'^{2}{}_{\nu} &\to W^{+}_{\nu},\\ \bar{A}'^{3}{}_{\nu} &\to Z_{\nu},\\ \bar{A}'^{4}{}_{\nu} &\to A_{\nu}.\end{aligned} \tag{39}$$

$\bar{B}^{a}{}_{\mu}$ will be responsible for the mass generation and, if so wished, for the introduction of a scalar field, a candidate Higgs field.

The mass terms are obtained by exploring the extended gauge dynamics of GA. The equation of motion is now

$$\bar{X}'_{\mu} \bar{F}'^{a\mu\nu} - \bar{C}^{a}{}_{d\mu} \bar{F}'^{d\mu\nu} = 0, \tag{40}$$

with $\bar{X}'_{\mu} = \partial_{\mu} - \bar{A}'^{a}{}_{\mu} \bar{X}_{a}$ and $\bar{X}_{a} \bar{A}'^{b}{}_{\mu} = \bar{f}^{b}{}_{ca} \bar{A}'^{c}{}_{\mu} - \bar{C}^{b}{}_{a\mu}$. This expression can be rewritten with the help of (15) as

$$\partial_{\mu} \bar{F}'^{a\mu\nu} - \bar{f}^{a}{}_{cb} \bar{F}'^{c\mu\nu} \bar{A}'^{b}{}_{\mu} + \bar{A}'^{b}{}_{\mu} \bar{R}'^{a}{}_{b^{\mu\nu}} - \bar{C}^{a}{}_{d\mu} \bar{F}'^{d\mu\nu} = 0,$$

with

$$\bar{F}'^{a}{}_{\mu\nu} = \partial_{\mu} \bar{A}'^{a}{}_{\nu} - \partial_{\nu} \bar{A}'^{a}{}_{\mu} + \bar{f}^{a}{}_{bc} \bar{A}'^{b}{}_{\mu} \bar{A}'^{c}{}_{\nu} - \bar{C}^{a}_{c\mu} \bar{A}'^{c}{}_{\nu} + \bar{C}^{a}{}_{c\nu} \bar{A}'^{c}{}_{\mu}, \tag{41}$$

and

$$\bar{R}'^{a}{}_{b}{}^{\mu\nu} = \bar{X}'^{\mu} \bar{C}^{a}{}_{b}{}^{\nu} - \bar{X}'^{\nu} \bar{C}^{a}{}_{b}{}^{\mu} - \bar{C}^{a}{}_{e}{}^{\mu} \bar{C}^{e}{}_{b}{}^{\nu} + \bar{C}^{a}{}_{e}{}^{\nu} \bar{C}^{e}{}_{b}{}^{\mu}. \tag{42}$$

We shall henceforth simplify the notation to agree with that of section 2, dropping the bars from fields and structure constants. In terms of the broken potentials $A'^{a}{}_{\mu}$, Eq. (40) can be rewritten as

$$\begin{aligned}&\partial^{\mu} \partial_{\mu} A'^{a\nu} - \partial_{\mu} \partial^{\nu} A'^{a\mu} + \left[f^{a}{}_{bc} A'^{c\nu} + C^{a}{}_{b}{}^{\nu} \right] \partial_{\mu} A'^{b\mu}\\ &+ \left[2 f^{a}{}_{bc} A'^{b}{}_{\mu} - 2C^{a}{}_{c}{}^{\mu} \right] \partial_{\mu} A'^{c\nu} + \left[\partial_{\mu} C^{a}{}_{c}{}^{\nu} - C^{a}{}_{d\mu} C^{d}{}_{c}{}^{\nu} \right] A'^{c\mu}\\ &+ A'^{b}{}_{\mu} R'^{a}{}_{b}{}^{\mu\nu} + \left[C^{a}{}_{c\mu} + f^{a}{}_{cb} A'^{b}{}_{\mu} \right] \partial^{\nu} A'^{c\mu}\\ &+ \left[f^{a}{}_{cb} C^{c}{}_{d\mu} - f^{c}{}_{bd} C^{a}{}_{c\mu} \right] A'^{b\mu} A'^{d\nu} - f^{a}{}_{cb} C^{c}{}_{d}{}^{\nu} A'^{b}{}_{\mu} A'^{d\mu}\\ &- f^{a}{}_{cb} f^{c}{}_{de} A'^{b}{}_{\mu} A'^{d\mu} A'^{e\nu} + \left[-\partial_{\mu} C^{a\mu}_{d} + C^{a}{}_{c\mu} C^{c}{}_{d}{}^{\mu} \right] A'^{d\nu} = 0.\end{aligned} \tag{43}$$

For $C = 0$ this gives just the Yang–Mills equations. Let us therefore examine the nontrivial part, with the terms containing $C's$:

$$
\begin{aligned}
&C^a{}_b{}^\nu \partial_\mu A'^{b\mu} - 2C^a{}_{c\mu}\partial_\mu A'^{c\nu} + C^a{}_{c\mu}\partial^\nu A'^{c\mu} \\
&\quad + \left[\partial_\mu C^a{}_c{}^\nu - C^a{}_{d\mu}C^d{}_c{}^\nu + R'^a{}_{c\mu}{}^\nu\right] A'^{c\mu} \\
&\quad + \left[f^a{}_{cb}C^c{}_{d\mu} - f^c{}_{bd}C^a{}_{c\mu}\right] A'^{b\mu}A'^{d\nu} - f^a{}_{cb}C^c{}_d{}^\nu A'^b{}_\mu A'^{d\mu} \\
&\quad + \left[-\partial_\mu C^a{}_d{}^\mu + C^a{}_{c\mu}C^c{}_d{}^\mu\right] A'^{d\nu}.
\end{aligned}
\tag{44}
$$

We see that the last two terms, which we indicate

$$\mathbf{Q}^{a\nu} = \left[-\partial_\mu C^a{}_d{}^\mu + C^a{}_{c\mu}C^c{}_d{}^\mu\right] A'^{d\nu}, \tag{45}$$

can provide a mass term for the component a of A', given by the term with $d = a$ of the sum over d. In fact $\mathbf{Q}^{a\nu}$, besides being responsible for the masses, will give rise to coupling terms. As our main purpose is to obtain the phenomenologically correct values for the masses, we begin by analyzing (45) for each component of $A'^{d\nu}$.

An initial point to consider is the experimental fact that one of the neutral bosons remains massless, that is, it behaves like a connection. This means that the C related to it must be zero. In fact, up to this point, there is no difference between $A'^0{}_\nu$ (or A^ν) and $A'^3{}_\nu$ (or Z^ν). We shall make the choice $C^0{}_{a\mu} \equiv 0$, which will break the symmetry between them. Thus, $A'^{0\nu} = A^\nu$ will be the photon field. Its transformation under the Glashow group is

$$X_a A'^{0\nu} = f^0{}_{ca}A'^{c\nu}, \tag{46}$$

which means

$$C^0{}_{c\mu} \equiv 0, \quad \forall\, c \text{ and } \mu. \tag{47}$$

It also implies $B^0{}_\mu \equiv 0$.

Electric charge conservation imposes additional conditions. A term by term examination of Eq. (44) for each component shows that many components of C must be made to vanish. The only nonvanishing components are four: $C^1{}_{1\mu}$, $C^2{}_{2\mu}$, $C^3{}_{0\mu}$, and $C^3{}_{3\mu}$. Up to this point these $C's$ are arbitrary but, as we are going to see, they will have to assume some special forms in order to generate the correct mass values.

Let us proceed to analyze $\mathbf{Q}^{a\nu}$, keeping only the four components of C above. Writing

$$\mathbf{Q}^{a\nu} = Q^a{}_d A'^{d\nu} \tag{48}$$

with

$$Q^a{}_d = \left[-\partial_\mu C^a{}_d{}^\mu + C^a{}_{c\mu}C^c{}_d{}^\mu\right], \tag{49}$$

we obtain:

Boson	$\mathbf{Q}^{a\nu}$
W^-_μ	$\mathbf{Q}^{1\nu} = \left(-\partial_\mu C^1{}_1{}^\mu + C^1{}_{1\mu} C^1{}_1{}^\mu\right) W^{-\nu}$
W^+_μ	$\mathbf{Q}^{2\nu} = \left(-\partial_\mu C^2{}_2{}^\mu + C^2{}_{2\mu} C^2{}_2{}^\mu\right) W^{+\nu}$
Z_μ	$\mathbf{Q}^{3\nu} = \left(-\partial_\mu C^3{}_3{}^\mu + C^3{}_{3\mu} C^3{}_3{}^\mu\right) Z^\nu - \partial_\mu C^3{}_{0\mu} A^\nu$
A_μ	$\mathbf{Q}^{0\nu} = 0$

The masses mw and mz of $W^-(W^+)$ and Z are known. To match their values we must choose some model for the C's.

4.2. Restricting the Model

Our model aims to recover all the predictions of W-S theory without remaining restricted to them. This means that we leave it open to the possibility of new couplings. It is crucial to remember that C accounts also for the noncovariance of B,

$$C^a{}_{b\mu} = -X_b\left(B^\alpha{}_\mu\right) + f^a{}_{cb} B^c{}_\mu \tag{50}$$

with B arbitrary. We adopt for $B^c{}_\mu$ an expression as general as possible at this point, also contemplating simplicity. It must contain one term independent of the coordinates x^μ to originate masses, and a linear part that will be related to the candidate Higgs field. We write then

$$B^a{}_\mu = \alpha M^\alpha{}_\mu + \beta K^a{}_\mu(x^\mu), \tag{51}$$

with α, β real numbers. Substituting (51) in (50) and considering the structure constants (32) we get

$$\begin{aligned}
C^1{}_{1\mu} &= -\alpha X_1\left(M^1{}_\mu\right) - \beta X_1\left(K^1{}_\mu\right) - i\cos\theta\left[\alpha M^3{}_\mu + \beta K^3{}_\mu\right],\\
C^2{}_{2\mu} &= -\alpha X_2\left(M^2{}_\mu\right) - \beta X_2\left(K^2{}_\mu\right) + i\cos\theta\left[\alpha M^3{}_\mu + \beta K^3{}_\mu\right],\\
C^3{}_{3\mu} &= -\alpha X_3\left(M^3{}_\mu\right) - \beta X_3\left(K^3{}_\mu\right),\\
C^3{}_{0\mu} &= -\alpha X_0\left(M^3{}_\mu\right) - \beta X_0\left(K^3{}_\mu\right)
\end{aligned} \tag{52}$$

and we can evaluate $\boldsymbol{Q}^{a\nu}$ for each particle.

4.2.1. Photon

Since we have in this case a connection, $B^0{}_\mu \equiv 0$ and $C^0{}_{a\mu} \equiv 0$. In consequence,

$$\mathbf{Q}^{a\nu}{}_{\text{photon}} = 0, \tag{53}$$

and there is no mass generation for the photon.

4.2.2. Z

For the third component, we have

$$\mathbf{Q}^{3\nu}{}_{Z} = Q^3{}_3 Z^\nu + Q^3{}_0 a^\nu \tag{54}$$

with

$$Q^3{}_3 = \alpha^2 X_3\left(M^3{}_\mu\right) X_3(M^{3\mu}) + 2\alpha\beta X_3\left(M^3{}_\mu\right) X_3(K^{3\mu}) \\ + \beta^2 X_3\left(K^3{}_\mu\right) X_3(K^{3\mu}) + \beta X_3\left(\partial_\mu K^{3\mu}\right), \tag{55}$$

and

$$Q^3{}_0 = \beta X_0(\partial_\mu K^{3\mu}). \tag{56}$$

The first term in (55) must be the mass term up to a sign. We must have then

$$\alpha^2 X_3\left(M^3{}_\mu\right) X_3(M^{3\mu}) = -m_Z^2. \tag{57}$$

One possible solution comes from the condition

$$X_3\left(M^3{}_\mu\right) = \pm\frac{i}{2\alpha} m_Z I_\mu, \tag{58}$$

where I_μ is a row-vector satisfying $I_\mu I^\mu = 4$.

The term quadratic in β of Eq. (55) corresponds to an interaction of the field Z with a field $\sigma(x)$ up to a constant D:

$$\beta^2 X_3\left(K^3{}_\mu\right) X_3(K^{3\mu}) = D^2\sigma^2, \tag{59}$$

which leads to the condition

$$X_3(K^{3\mu}) = \pm\frac{D\sigma}{\beta} I^\mu. \tag{60}$$

Finally, using conditions (58) and (60), the second and fourth terms in (55) are obtained. The second term is

$$2\alpha\beta X_3\left(M^3{}_\mu\right) X_3(K^{3\mu}) = 2iD\sigma m_Z. \tag{61}$$

It is remarkable that the two terms (57) and (59) imply, in our formalism, the presence of (61), a type of coupling which is also present in the W-S model. The fourth term corresponds to

$$\beta X_3\left(\partial_\mu K^{3\mu}\right) = \pm D(\partial_\mu\sigma) I^\mu, \tag{62}$$

which shows that our model contains a new coupling, absent in the W-S model: A derivative σ field term coupled with Z^ν.

It is possible to choose D so that we have the same couplings of W-S with their constants. Let us begin by matching the linear coupling in Eq. (61)

$$2iD\sigma m_Z = -\frac{g\sigma m_{\mathrm{Z}}}{\cos\theta_{\mathrm{W}}} \tag{63}$$

we then have

$$D = \frac{ig\sigma}{2\beta \cos\theta_{\mathrm{W}}} \tag{64}$$

where $g = e \sin\theta_{\mathrm{W}}$ is a coupling constant, θ_{W} is the Weinberg angle, and σ represents the Higgs field.

Taking Eq. (64) into (59), we obtain exactly the Z–σ^2 interaction term of the W-S model,

$$\beta^2 X_3\left(K^3{}_\mu\right) X_3(K^{3\mu}) = -\frac{g^2\sigma^2}{4\cos\theta_{\mathrm{W}}}, \tag{65}$$

The term (62) becomes:

$$\beta X_3(\partial_\mu K^{3\mu}) = \frac{ig}{2\cos\theta_{\mathrm{W}}}(\partial_\mu\sigma)I^\mu, \tag{66}$$

the above mentioned nonstandard term in our model. Here, the question arises whether the mixing angle introduced by the structure constants of GA coincides with the Weinberg angle. The answer is positive. The equality is necessary if we want to match the coupling terms.

Summing up our results, we have obtained in the field equation the following terms:

$$\mathbf{Q}^{3\nu}{}_Z = \left[-m_{\mathrm{Z}}^2 - \frac{g^2\sigma^2}{4\cos\theta_{\mathrm{W}}} - \frac{g\sigma m_{\mathrm{Z}}}{\cos\theta_{\mathrm{W}}} + \frac{ig}{2\cos\theta_{\mathrm{W}}}(\partial_\mu\sigma)I^\mu\right] Z^\nu$$
$$+ \beta X_0(\partial_\mu K^{3\mu})A^\nu. \tag{67}$$

This coincides with the W-S model except for the presence of the last two terms. These are theoretically consistent within the model, but their eventual measurable effects are still to be evaluated.

4.2.3. W^- and W^+

Once we have learned that it is possible to recover the W-S model by choosing correctly the free parameters of our model, we make from now on direct contact with that model. As in the previous case, we write

$$\mathbf{Q}^{1\nu}{}_{W^-} = Q^1{}_1 W^{-\nu} \tag{68}$$

with

$$Q^1{}_1 = \alpha^2\left[X_1\left(M^1{}_\mu\right) + i\cos\theta_{\mathrm{W}} M^3{}_\mu\right]^2 + 2\alpha\beta\left\{X_1\left(M^1{}_\mu\right)X_1(K^{1\mu})\right.$$
$$+ i\cos\theta_{\mathrm{W}}\left[K^3{}_\mu X_1(M^{1\mu}) + M^3{}_\mu X_1(K^{1\mu}\right] - \left.\cos^2\theta_{\mathrm{W}} M^3{}_\mu K^{3\mu}\right\}$$
$$+ \beta^2\left[X_1\left(K^1{}_\mu\right) + i\cos\theta_{\mathrm{W}} K^3{}_\mu\right]^2 + \beta\left[X_1\left(\partial_\mu K^{1\mu}\right) + i\cos\theta_{\mathrm{W}}\partial_\mu K^{3\mu}\right].$$

For the mass term we have

$$-\alpha^2 \left[\cos\theta_\mathrm{W} M^3{}_\mu - i X_1 \left(M^1{}_\mu\right)\right]^2 = -m_\mathrm{W}^2,$$

corresponding to

$$X_1 \left(M^1{}_\mu\right) = \pm \left(\frac{im_\mathrm{W}}{2\alpha} I_\mu\right) - i \cos\theta_\mathrm{W} M^3{}_\mu. \tag{69}$$

The term quadratic in β matched to W-S coupling leads to

$$-\beta^2 \left[\cos\theta_\mathrm{W} K^3{}_\mu - i X_1 \left(K^{\mathbf{1}}{}_\mu\right)\right]^2 = -\frac{1}{4} g^2 \sigma^2,$$

and we have the condition

$$X_1 \left(K^1{}_\mu\right) = \pm \left(\frac{ig\sigma}{4\beta} I_\mu\right) - i \cos\theta_\mathrm{W} K^3{}_\mu. \tag{70}$$

It is important to notice that the signs in Eqs. (69) and (70) are defined independently from each other. Thus we may fit them to obtain from the term proportional to $\alpha\beta$ the linear term in the Higgs field present in the W-S model:

$$\begin{aligned} 2\alpha\beta \big[X_1 \left(M^1{}_\mu\right) X_1(K^{1\mu}) + i \cos\theta_\mathrm{W} K^3{}_\mu X_1(M^{1\mu}) \\ + i \cos\theta_\mathrm{W} M^3{}_\mu X_1(K^{1\mu}) - \cos^2\theta_\mathrm{W} M^3{}_\mu K^{3\mu}\big] = -g m_\mathrm{W} \sigma \end{aligned}$$

with

$$X_1 \left(M^1{}_\mu\right) = \frac{im_\mathrm{W}}{2\alpha} I_\mu - i \cos\theta_\mathrm{W} M^3{}_\mu \tag{71}$$

and

$$X_1 \left(K^1{}_\mu\right) = \frac{ig\sigma}{4\beta} I_\mu - i \cos\theta_\mathrm{W} K^3{}_\mu. \tag{72}$$

As in the previous case, we get also an extra term:

$$\beta \left[X_1 \left(\partial_\mu K^{1\mu}\right) + i \cos\theta_\mathrm{W} \left(\partial_\mu K^{3\mu}\right)\right] = \frac{ig}{2} (\partial_\mu \sigma) I^\mu.$$

Gathering terms we obtain

$$\mathbf{Q}^{1\nu}{}_{\mathrm{W}^-} = \left[-m_\mathrm{W}^2 - \frac{1}{4} g^2 \sigma^2 - g m_\mathrm{W} \sigma + \frac{ig}{2} (\partial_\mu \sigma) I^\mu\right] W^{-\nu}, \tag{73}$$

possessing the mass term, the expected coupling with the Higgs field, and a new, nonstandard derivative coupling.

In the same way, evaluating $\mathbf{Q}^{2\nu}{}_{W^+}$, we obtain the following conditions:

$$X_2 \left(M^2{}_\mu\right) = -\frac{im_\mathrm{W}}{2\alpha} I_\mu + i \cos\theta_\mathrm{W} M^3{}_\mu \tag{74}$$

and

$$X_2\left(K^2{}_\mu\right) = -\frac{ig\sigma}{4\beta} I_\mu + i\cos\theta_{\rm W} K^3{}_\mu, \tag{75}$$

leading to

$$\mathbf{Q}^{2\nu}{}_{\rm W^+} = \left[-m_{\rm W}^2 - \frac{1}{4}g^2\sigma^2 - g m_{\rm W}\sigma - \frac{ig}{2}(\partial_\mu\sigma)I^\mu\right] W^{+\nu}. \tag{76}$$

The signs of Eqs. (74) and (75) are chosen, on one hand, to fit those of W-S model; on the other hand, to be consistent with the equality of the W^+ and W^- masses. This implies, from the definition of $\mathbf{Q}^{2\nu}$ in the table below Eq. (49) and from the expressions of $C^1{}_{1\mu}$ and $C^2{}_{2\mu}$ in (52), that we must have

$$C^1{}_{1\mu} = -C^2{}_{2\mu}. \tag{77}$$

A positive sign in the left-hand side would lead to $C^3{}_{3\mu} = 0$.

For completeness, we exhibit the components of C determined by the model:

$$C^1{}_{1\mu} = -C^2{}_{2\mu} = -\frac{i}{2}\left[m{\rm w} + \frac{g\sigma}{2}\right] I_\mu,$$

$$C^3{}_{3\mu} = -\frac{i}{2}\left[m{\rm z} + \frac{g\sigma}{2\cos\theta{\rm w}}\right] I_\mu.$$

Notice that, using the relation $m{\rm z} = m{\rm w}/\cos\theta{\rm w}$, we find $C^3{}_{3\mu} = C^1{}_{1\mu}/\cos\theta{\rm w}$. The component $C^3{}_{0\mu}$ is up to now completely arbitrary. Loosely speaking, C, the object which measures the covariance breaking of A'_μ is directly related to mass generation and to the existence of another field which we are associating to the Higgs field.

A balance of the degrees of freedom should be done. Firstly, we notice that in the very beginning of the process of adding a noncovariant part to the connection, we have three degrees of freedom for $B^a{}_\mu$. They come from the three nonnull gauge components, each one with two degrees of freedom (it is a massless vector term) from which we subtract three degrees of freedom due to the constraints (13) on $B^a{}_\mu$. Now, adding the eight degrees of freedom for the massless fields $A^a{}_\mu$ of the theory, it totals 11. The same total number of degrees of freedom is computed after the process of mass generation, since we have three massive $A'^c{}_\mu$, amounting to nine degrees of freedom, plus the boson that remains massless, with two degrees of freedom. Notice finally that the degrees of freedom corresponding to the candidate Higgs field σ are already included in those of B's.

5. CONCLUSIONS AND FINAL COMMENTS

We have presented a procedure to generate masses for the bosons in electroweak theory which is alternative to spontaneous symmetry breaking. The method takes its roots in the theory of Lie algebra extensions, applied in the case to the Glashow algebra. The extension of a Lie algebra is another Lie algebra, so that new Jacobi identities appear. One of them leads to a new Bianchi identity. The dynamic equations for the boson fields are obtained by applying the duality prescripition to that Bianchi identity. The formalism leads, in this way, directly to the field equations. It should be recalled that quantization, despite the modern heavy reliance on Lagrangians and some statements to the contrary, can be realized directly from the field equations (Aldrovandi and Kraenkel, 1989; Bjorken and Drell, 1964; Kallen, 1950; Yang and Feldman, 1950).

Working only with the equations of motion, we have shown that it is possible to obtain the correct masses for W^+, W^-, and Z, while keeping a fourth boson A massless. The model predicts all the bosonic couplings present in W-S model.

Another feature of our model is the introduction of a scalar field σ, candidate (so called because its dynamics is still under examination) to play the role of the Higgs field of the W-S model. Besides the Higgs–boson couplings of the W-S model, four nonstandard couplings turn up. The latter are consistent within the model and their contributions to cross sections are under study. The presence of σ field is necessary to have the same number of the degrees of freedom before and after mass generation. It is also directly linked to the coefficients $C^a{}_{b\mu}$, which measure the direct product breaking responsible for the appearence of the masses.

The σ field dynamics, the renormalizability of the extra couplings, as well as the Lagrangian formalism, are still under study. The same is true of the gravitational counterpart of the model (Aldrovandi *et al.*, in preparation).

ACKNOWLEDGMENTS

The authors thank FAPESP (Brazil) and CNPq (Brazil) for financial support. They also thank B. M. Pimentel for very fruitful discussions.

REFERENCES

Aldrovandi, R. (1991a). *Journal of Mathematical Physics* **32**, 2503.

Aldrovandi, R. (1991b). *Physics Letters A* **l55**, 459.

Aldrovandi, R., Andrade, V. C., Barbosa, A. L., and Pereira, J. G. (in preparation). *Gravitational Model from Extended Gauge Theories*.

Aldrovandi, R. and Barbosa, A. L. (2000). *International Journal of Theoretical Physics* **39**, 2779.

Aldrovandi, R. and Kraenkel, R. (1989). *Journal of Mathematical Physics* **30**, 1866.

Aldrovandi, R. and Pereira, J. G. (1995). *An Introduction to Geometric Physics*, World Scientific, Singapore.

Barbosa, A. L. (2000). *International Journal of Theoretical Physics* **39**, 1985.

Bjorken, J. D. and Drell, S. D. (1964). *Relativistic Quantum Mechanics*, McGraw-Hill, New York.

Cheng, T. P. and Li, L. F. (1984). *Gauge Theory of Elementary Particles*, Oxford University Press, Oxford.

Cho, Y. M. (1975). *Physical Review D: Particles and Fields* **12**, 3789.

Greiner, W. and Müller, B. (1996). *Gauge Theory of Weak Interactions*, Springer, New York.
Källen, G. (1950). *Arkiv foer Fysik* **2**, 187 and 371.
Kobayashi, S. and Nomizu, K. (1963). *Foundations of Differential Geometry, Vol. 1*, Interscience, New York.
Mandl, F. and Shaw, G. (1984). *Quantum Field Theory*, Wiley, New York.
Nakahara, M. (1990). *Geometry, Topology and Physics*, IOP Publishing, Bristol.
Popov, D. A. (1975). *Theoretical and Mathematical Physics* **24**, 347.
Yang, C. N. and Feldman, D. (1950). *Physical Review* **79**, 972.